$\therefore$ (*a*) would be the better investment.

$120\times\$13.50=\$1620$; $\$1620\times.10\frac{1}{2}\times\frac{1}{12}=\$14.175=$ profit of first investment over second.

2. (*a*) Let $\$13.50=p$, $.00875=r$, $\$1000=a$; then $(a-p)=$ amount due first of first month; $(a-p)(1+r)-p=$ amount due first of second month; $[(a-p)(1+r-p)](1+r)-p=$ amount due first of third month; $\{[(a-p)(1+r)-p)](1+r)-p\}(1+r)-p=$ amount due first of fourth month.

$\therefore$ for any month, as 10,
$\{[(a-p)(1+r)-p](1+r)-p](1+r)-p\}\ [(a-p)(1+r)-p)(1+r)-p^7(1+r)-p]$.

2. $a(1+r)-p$ at end of first month; $[a(1+r)-p](1+r)-p$ at end of second month; $\{[a(1+r)-p](1+r)-p\}(1+r)-p$ at end of third month. At end of tenth month, $\langle\ \{[a(1+r)-p](1+r)-p\}^8(1+r)-p\ \rangle$.

The solution by Dr. Zerr seems to us to be a proper disposition of the problem, but we publish the other solutions for comparison.

Reply to Note of Dr. Drummond on No. 78.—Its one *real* point is, that my solution, on page 43, line 5, omits to denote $-xy=-6$, as $-x_1y_1=-6$, (which please correct). Thus result $x^4-8x^2+16=0$; $y^4-18y^2+81=0$, whose roots are: $x=2$ to $y=3$; $x_a=-2$ to $y_a=-3$, of xy in (I)(II); and also $x_1=2_1$ to $y_1=3_1$; $x_2=-2_1$ to $y_2=-3_1$ (by last two clauses on page 44), the factors of $-x_1y_1=-6_1$ in parity to $x_{11}y_{11}=35$......(*D*) on page 44. Or if the objector insist,—of $x_1y_1=6_1$ and $x_ay_a=6_a$ reading then $xy=6$ and $x_ay_a=6_a$. No other error is specified or shown. As the equations cited do not occur in my solution,—for his $x^2=36$ is quadratic while $x^2y^2=36$ from (I)(II) is bi-quadratic; I need only say that the roots of his $x^4=1296$ are 6; -6, $6\sqrt{(-1)}$; $-6\sqrt{(-1)}$, not ±6; ∓6. Roots $x^4=16$ are 2; -2; $2\sqrt{(-1)}$; $-2\sqrt{(-1)}$ and not ±2; ∓2 as he mistakes me to mean.

J. M. BOORMAN.

GEOMETRY.

99. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Find the locus of the vertices of all right cones which have the same given ellipse as a base.

Solution by the PROPOSER.

Any generator, $x=a'z+\alpha$......(1). $y=b'z+\beta$......(2), must satisfy
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$......(3), and $z=0$......(4), giving $\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}=1$......(5).

If the vertex be (x', y', z'), (1) and (2) must be satisfied, and we have $x'=a'z'+\alpha$......(6), $y'=b'z'+\beta$......(7).

Eliminating a', b', α, β from (1), (2), (6), (7),

$$\frac{1}{a^2}(z'x-x'z)^2+\frac{1}{b^2}(z'y-y'z)^2=(z'-z)^2\ .....\ (8),$$

the equation to a cone having (x', y', z') for vertex, and (3) for base.

$$(8) \text{ is } \frac{z'^2}{a^2}x^2+\frac{z'^2}{b^2}y^2+\left(\frac{x'^2}{a^2}+\frac{y'^2}{b^2}-1\right)z^2-\frac{2y'z'}{b^2}yz-\frac{2x'z'}{a^2}xz+2z'z-z'^2=0 \;..(9).$$

The conditions for (9) to be a cone of revolution are $x'=0$,

$$\frac{z'^2}{a^2}-\frac{y'^2}{b^2-a^2}=1\ldots\ldots(10),$$

an hyperbola for the required locus.

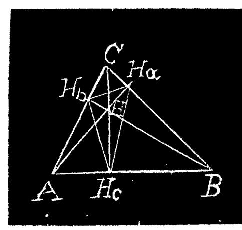
Also solved by *G. B. M. ZERR.*

100. Proposed by CHARLES CARROLL CROSS, Libertytown, Md.

O, O_1, O_2, O_3 are the centers of the inscribed and three escribed circles of a triangle ABC. Prove $AO.AO_1.AO_2.AO_3=AB^2.AC^2$.

I. Solution by the PROPOSER.

Consider the ex-central triangle ABC as the original triangle. Then $H_aH_bH_c$ is the pedal triangle, and the incenter O becomes the orthocenter H.



Hence we have to prove $AH_c\times BH_c\times CH_c\times HH_c=H_aH_c^2\times H_bH_c^2$.

We readily find by trigonometry that

$$AH_c=\frac{b^2+c^2-a^2}{2c},\ BH_c=\frac{a^2+c^2-b^2}{2c},$$

$$CH_c=\frac{\sqrt{[4a^2c^2-(a^2+c^2-b^2)^2]}}{2c}=\frac{2\triangle}{c},\quad HH_c=\frac{(b^2+c^2-a^2)(a_2+c^2-b^2)}{8c\triangle},$$

$$H_aH_c=\frac{b(a^2+c^2-b^2)^2}{2ac},\ H_bH_c=\frac{a(b^2+c^2-a^2)}{2bc}.$$

Substituting in the problem we have

$$\frac{b^2+c^2-a^2}{2c}\times\frac{a^2+c^2-b^2}{2c}\times\frac{2\triangle}{c}\times\frac{(b^2+c^2-a^2)(a^2+c^2-b^2)}{8c\triangle}$$

$$=\frac{b^2(a^2+c^2-b^2)^2}{4a^2c^2}\times\frac{a^2(b^2+c^2-a^2)^2}{4b^2c^2}.$$

Since these equations cancel, the proposition is proved.

Mr. Cross should have been credited with solutions of problems 96 and 98 in Geometry, and 100 and 101 in Arithmetic.